Using complete sentences and proper mathematical notation, write the formal definition of "continuous (at a point)". **SCORE: _____ / 2 PTS**

f IS CONTINUOUS AT a IFF $f(a)$ EXISTS, $\lim_{x\to a} f(x)$ EXISTS AND $\lim_{x\to a} f(x) = f(a)$

GRADED BY ME

Using complete sentences and proper mathematical notation, write the formal definition of "derivative (function)". **SCORE: _____ / 1 PT**

THE DERIVATIVE OF f IS $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

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The amount of sugar that can be dissolved in a cup of coffee depends on the temperature of the coffee. **SCORE: _____ / 2 PTS**

Suppose $s = f(t)$, where s is the amount of sugar that can be dissolved (in grams), and t is the termperature (in degrees Celsius).

[a] What does $f(80) = 60$ mean ? Give the correct units for all numbers in your answer.

60g OF SUGAR CAN BE DISSOLVED IN A CUP OF 80°C COFFEE

[b] What does $f'(80) = 2$ mean ? Give the correct units for all numbers in your answer.

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IF A CUP OF COFFEE IS 80°C, $\approx$2 ADDITIONAL GRAMS OF SUGAR CAN BE DISSOLVED IN IT FOR EACH 1°C HOTTER THE COFFEE GETS.

Find the following limits. **SCORE: _____ / 7 PTS**

Each answer should be a number, ∞, $-\infty$, or DNE (only if the other answers do not apply).

[a] $\lim_{x\to\infty} \tan^{-1} x$

$= \boxed{\frac{\pi}{2}}$ 1

[b] $\lim_{x\to\infty} \arccos e^{-x}$

$= \boxed{\arccos 0}$ 1

$= \boxed{\frac{\pi}{2}}$ 1

[c] $\lim_{x\to-\infty} \frac{\sqrt{16x^2-25x}}{3-7x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$

$= \lim_{x\to-\infty} \frac{\sqrt{16x^2-25x} \cdot -\sqrt{\frac{1}{x^2}}}{\frac{3}{x}-7} = \lim_{x\to-\infty} \boxed{\frac{-\sqrt{16-\frac{25}{x}}}{\frac{3}{x}-7}} = \frac{-\sqrt{16-0}}{0-7} = \boxed{\frac{4}{7}}$ 1

1 2

Prove that $\ln x = \frac{1}{x}$ for some x in the interval $(1, e)$. **DO NOT ATTEMPT TO SOLVE FOR *x*.** **SCORE: _____ / 4 PTS**

LET $f(x) = \ln x - \frac{1}{x}$. f IS CONTINUOUS ON $[1, e]$ SINCE IT IS THE DIFFERENCE OF CONTINUOUS FUNCTIONS (1)

$f(1) = \ln 1 - \frac{1}{1} = -1$ AND $f(e) = \ln e - \frac{1}{e} = 1 - \frac{1}{e} = \frac{e-1}{e}$

$-1 < 0 < \frac{e-1}{e}$, SO BY IVT FOR SOME $c \in (1, e)$, $f(c) = 0$

(1) (1) IE. $\ln c - \frac{1}{c} = 0$ OR $\ln c = \frac{1}{c}$ (1)

Let $f(x) = \sqrt{19 - 3x}$. **SCORE: _____ / 8 PTS**

[a] Find $f'(x)$.

(1)

$$\lim_{h \to 0} \frac{\sqrt{19-3(x+h)} - \sqrt{19-3x}}{h} \cdot \frac{\sqrt{19-3(x+h)} + \sqrt{19-3x}}{\sqrt{19-3(x+h)} + \sqrt{19-3x}}$$

$$= \lim_{h \to 0} \frac{19-3(x+h)-(19-3x)}{h(\sqrt{19-3(x+h)} + \sqrt{19-3x})}$$

(2)

$$= \lim_{h \to 0} \frac{-3\cancel{h}}{\cancel{h}(\sqrt{19-3(x+h)} + \sqrt{19-3x})} = \frac{-3}{2\sqrt{19-3x}}$$ (1)

[b] Find the slope-point form of the equation of the tangent line to the curve of $f(x)$ at the point where $x = 1$.

$f'(1) = \frac{-3}{2\sqrt{16}} = -\frac{3}{8}$ (1) $y - 4 = -\frac{3}{8}(x-1)$ (1)

[c] The position (in yards) of an object moving in a straight line is given by $s(t) = \sqrt{19 - 3t}$, where t is the time in minutes. Find the instantaneous velocity of the object at time $t = 5$. Give the correct units for your answer.

$S'(5) = \frac{-3}{2\sqrt{4}} = -\frac{3}{4}$ (1) YARD/MINUTE (1)

Determine if each of the following functions is continuous. **STATE YOUR CONCLUSIONS CLEARLY.** **SCORE: _____ / 6 PTS**
If a function is continuous, justify your conclusion using the definition(s) and/or theorems.
If a function is not continuous, show clearly which part of the definition of "continuous" is not true.

[a] $f(x) = \begin{cases} \frac{x^3+1}{x^2-1}, & \text{if } x < -1 \\ \frac{x^2-4}{x+3}, & \text{if } x > -1 \end{cases}$

$f(-1)$ DNE
f NOT CONT. (1)

[b] $f(x) = \begin{cases} x^3 - 2x^2 - 8, & \text{if } x \le 3 \\ x^3 - 4x^2 + 10, & \text{if } x > 3 \end{cases}$

(1) f IS CONT AT $x \neq 3$ (POLYNOMIAL)

$f(3) = 3^3 - 2(3)^2 - 8 = 1$

($\frac{1}{2}$) $\lim_{x \to 3^+} (x^3 - 4x^2 + 10) = 3^3 - 4(3)^2 + 10 = 1$

($\frac{1}{2}$) $\lim_{x \to 3^-} (x^3 - 2x^2 - 8) = 3^3 - 2(3)^2 - 8 = 1$

(1)

(1) $\lim_{x \to 3} f(x) = 1 = f(3) \rightarrow f$ IS CONT. AT $x=3$

$\rightarrow f$ IS CONT. (1)